TARIFF AGREEMENTS AND NON-RENEWABLE RESOURCE INTERNATIONAL MONOPOLIES: PRICES VERSUS QUANTITIES*

Santiago J. Rubio**

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** Department of Economic Analysis, University of Valencia, Edificio Departamental Oriental, Avda. de los Naranjos, s/n, 46022 Valencia, Spain, E-mail address: Santiago.Rubio@uv.es.

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ABSTRACT

In this paper we model the case of an international non-renewable resource monopolist as a differential game between the monopolist and the governments of the importing countries, and we investigate whether a tariff on the resource importations can be advantageous for the importing countries. We find that the results depend crucially on the kind of strategies the importing country governments can play and on whether the monopolist chooses the price or the extraction rate. For a price-setting monopolist it is shown that the importing countries cannot use a tariff to capture monopoly rents if they are constrained to use open-loop strategies, even if the governments sign a tariff agreement. This result is drastically modified if the importing countries in the tariff agreement use Markov (feedback) strategies. For a quantity-setting monopolist the nature of the game changes and an open-loop tariff is advantageous for the importing countries. Moreover, in this case the importing countries in a tariff agreement enjoy a strategic advantage which allows them to behave as a leader.

Keywords: tariffs, tariff agreements, non-renewable resources, depletion effects, price-setting monopolist, quantity-setting monopolist, differential games, open-loop strategies, linear strategies, Markov-perfect Nash equilibrium, Markov-perfect Stackelberg equilibrium.

JEL Classification Numbers: C73, D41, D42, F02, H20, Q38.

1 Introduction

The issue of using an import tariff to capture non-renewable resource rents was addressed sometime ago in a nice piece of work by Bergstrom (1982). In his paper, he shows that if all importing countries of a competitively supplied non-renewable resource select the same *ad valorem* tariff on the resource consumed at any time, the tariff is advantageous for the importing countries in the sense that they capture resource rents from the exporting countries. He characterizes the Nash equilibrium of the game among the importing countries by a simple rule relating the equilibrium *ad valorem* tariff to demand elasticities and market shares. In the second part of the paper, he argues that almost all the profits of a monopolist can be taxed away by the importing countries if they choose a sufficiently high tariff as long as the profit-maximizing level of the monopolist's price is independent of the tariff. These results are obtained for a Hotelling-type model with a costlessly extracted non-renewable resource. Later, several papers addressed this issue, see Brander and Djajic (1983), Karp (1984), Maskin and Newbery (1990), Karp and Newbery (1991, 1992).[1] Among them only Brander and Djajic (1983) and Karp (1984) have addressed this issue for the case of a non-renewable resource monopoly. Brander and Djajic develop their analysis in the context of a simple two-country general equilibrium model of trade in exhaustible resources where it is assumed that the resource is extracted costlessly and used as an essential input in the production of a homogeneous consumption good. They find that the country without resource has an incentive to impose a tariff so as to extract at least some of the available rent. The magnitude of the optimal tariff is found to be an increasing function of the relative size of the importing country and approaches the confiscatory level as the resource importing country becomes very large. In Karp the interaction between a monopolist and a single buyer is modeled as a Stackelberg game where the extraction cost is inversely related to the stock (depletion effects) and the buyer is the leader of the game. The buyer chooses a tariff and the monopolist the rate of extraction. He shows that the open-loop tariff is temporally inconsistent because of the stock-dependence of extraction costs. Besides, he proposes a method of obtaining temporally consistent strategies and concludes that the consistent tariff against the monopolist is in general

[1] A recent contribution by Hörner and Kamien (2004) shows that the intertemporal no-arbitrage condition that arises if the durable good monopolist seller can commit to a price path mirrors the intertemporal no-arbitrage condition if the monopsonist buyer of an exhaustible resource supplied by competitive sellers can commit to a profile of import tariffs. The time-consistency of the tariff in this case has been studied by Kemp and Long (1980) and Karp (1984, 1991).

not identically zero which implies that the consistent tariff allows the buyer to improve his position.[2]

In the first part of the paper we model the case of a price-setting monopolist selling to noncooperative consuming nations, first studied by Bergstrom (1982), as a *differential game*. In the second part, we analyze the case of a quantity-setting monopolist. Moreover, we also extend his analysis by assuming that extraction cost is directly related to accumulated extractions (depletion effects).

Contrary to the results obtained by Bergstrom (1982), we find that the profit-maximizing level of the monopolist's price is not independent of the tariff and that the open-loop Nash equilibrium tariff is zero. This result appears because the user cost for the importing countries is equal to the tariff times the monopoly price and because the extraction cost is supported by the monopolist. In this case the user cost for importing countries must increase at a rate equal to the interest rate but this is incompatible with the fact that ultimately the rent must be zero because of the economic exhaustion of the resource. We also find that this result applies for a *per unit* tariff and when the importing countries cooperate imposing the *same* tariff rate on the resource importations, i.e., when the importing country governments sign a *tariff agreement*, and independently of whether it is assumed that the countries are symmetric or not.

In order to clarify whether this result is a consequence of the equilibrium concept used to solve the game, we propose a differential game between a monopolist and a coalition of importing country governments for which it is possible to calculate the stationary linear Markov strategies. The solution to this game establishes that the importing countries can capture part of the monopoly's rents for their consumer using a tariff on the resource importations. Thus, we find that when the monopolist chooses a *pricing policy* the tariff is advantageous for the importing countries if the importing countries co-operate through a tariff agreement and the optimal policy is defined using a feedback strategy.

For the case of a quantity-setting monopolist, as the importing countries have no influence on the dynamics of the stock, the game among the importing countries becomes, in fact, a static game and given the influence of the tariff rate on the monopoly price the importing countries find it advan-

[2] He proposes an alternative method to the one developed by Simaan and Cruz (1973), see Prop.2 in Karp's paper. Simaan and Cruz's method is for the leader to treat the follower's dynamic programming equation as a constraint, and to solve his own problem using dynamic programing methods. This "backward" method of solution eliminates at each point all control rules that are not optimal given the state at the time, and results in consistent control rules.

tageous to set a tariff on the resource importations.[3] On the other hand, if the governments of the importing countries sign a tariff agreement it is pretty obvious that the open-loop Nash equilibrium policy is to choose, for a given extraction rate, a tariff such that the monopolist's price be zero, but then the monopoly is not interested in exploiting the resource so that finally the consumers of the importing countries are not going to enjoy any surplus. However, the coalition has another alternative since the importing country governments, in fact, enjoy a strategic advantage given that they can influence the monopoly price through the tariff. In other words, the importing country governments have another alternative because the coalition can behave as a leader.[4] To conclude the analysis we calculate, following the Simaan and Cruz's (1973) method, the stationary Markov-perfect Stackelberg equilibrium in linear strategies which guarantees the strong time consistency of the tariff, and we obtain that the importing countries are interested in setting a tariff that provides the monopoly with the possibility of obtaining a positive price with the aim of getting a positive surplus for their consumers. A policy that clearly is superior to the one derived from the open-loop Nash equilibrium.

The paper is organized as follows. In Section 2 the case of a price-setting monopolist is studied and the case of a quantity-setting monopolist is dealt with in Section 3. The comparison between the Markov-perfect Nash equilibrium of the differential game where the monopolist sets up the price and the Markov-perfect Stackelberg equilibrium where the monopolist sets up the extraction rate appears in Section 4. Conclusions and subjects for future research are presented in Section 5.

2 The Case of a Price-setting Monopolist

As in Bergstrom's (1982) analysis, we shall confine ourselves to a partial equilibrium model. Assuming that the representative consumer of the importing country acts as a price-taker agent, we can write the consumer's welfare function as $U_i(q_i) - p(1+\theta_i)q_i + R_i$, where $U_i(q_i)$ is the consumer's gross surplus, q_i the amount of the resource bought by the representative consumer of the importing country i, p the international price of the resource, θ_i the "ad valorem" tariff rate on the resource imports fixed by the government of the importing country i, and R_i a lump-sum transfer that the consumer receives

[3] This is the same result as the one obtained by Karp and Newbery (1991) when competitive suppliers move first in the game. However, we get it here as a consequence of the fact that the monopolist uses the extraction rate as a control variable.

[4] This is the hypothesis explored by Karp (1984).

from the government. Thus, the resource demand depends only on the consumer price: $U_i'(q_i) = p(1+\theta_i)$ and the demand function can be written as $q_i = D_i(p(1+\theta_i))$ with $D_i' < 0$ if the marginal utility is decreasing.[5] Thus the aggregate demand is $Q = \sum_{i=1}^{n} D_i(p(1+\theta_i))$.

The governments set the tariff with the aim of maximizing the discounted present value of the representative consumer's welfare. They reimburse tariff revenues as *lump-sum transfers*, so that finally the consumer's welfare does not depend on tariff revenues. The optimal time path for the tariff is thus given by the solution of the following optimal control problem

$$\max_{\{\theta_i\}} \int_0^{\infty} e^{-rt} \left(U_i(D_i(p(1+\theta_i))) - pD_i(p(1+\theta_i))\right) dt, \; i = 1, ..., n. \tag{1}$$

where r is the discount rate.

On the other side of the market we have a monopoly extracting the resource at an aggregate cost equal to $c(x)Q$, where $c(x)$ is the marginal extraction cost, with $c' > 0$ and $c'' \geq 0$, x stands for the accumulated extractions and Q for the current extraction rate of the resource. The objective of the monopoly is to define a price strategy that maximizes the present value of profits

$$\max_{\{p\}} \int_0^{\infty} e^{-rt} \left((p - c(x)) \sum_{i=1}^{n} D_i(p(1+\theta_i)) \right) dt. \tag{2}$$

Given the price the extraction rate is determined by the demand function so that the dynamics of the accumulated extractions is

$$\dot{x} = Q = \sum_{i=1}^{n} D_i(p(1+\theta_i)), \quad x(0) = x_0 \geq 0. \tag{3}$$

Both types of players face the same dynamic constraint and the optimal time paths for the tariff and the price are given by the solution of the *differential game* between the monopolist and the n importing countries defined by $(1) - (2) - (3)$.

2.1 The Open-loop Nash Equilibrium

First, we write the Hamiltonian associated to the optimal control problems of the importing countries.

[5] D_i' stands for the derivative of the demanded quantity with respect to the consumer price: $p(1+\theta_i)$.

$$H_i = U_i(D_i(p(1+\theta_i))) - pD_i(p(1+\theta_i)) + \lambda_i \sum_{j=1}^{n} D_j(p(1+\theta_j)),$$

which yields the following necessary conditions

$$p\theta_i = -\lambda_i, \quad \dot{\lambda}_i = r\lambda_i, \tag{4}$$

which establish that for a given value of the co-state variable the price is a *strategic substitute* of the tariff rate. Notice that now, as the extraction costs are supported directly by the monopolist, the tariff, $p\theta_i$, must increase at the interest rate.

For the monopolist the Hamiltonian is

$$H_M = (p - c(x) + \lambda_M) \sum_{i=1}^{n} D_i(p(1+\theta_i)),$$

and the necessary conditions are

$$\sum_{i=1}^{n} D_i + (p - c + \lambda_M) \sum_{i=1}^{n} (1+\theta_i) D_i' = 0, \tag{5}$$

$$\dot{\lambda}_M = r\lambda_M + c' \sum_{i=1}^{n} D_i, \tag{6}$$

where (5) is the *instantaneous reaction function* of the monopoly. By differentiation we can obtain that

$$\frac{\partial p}{\partial \theta_i} = -\frac{pD_i' + (p - c + \lambda_M)\,(D_i' + p(1+\theta_i)\,D_i'')}{2\sum_{i=1}^{n}(1+\theta_i)D_i' + (p - c + \lambda_M)\sum_{i=1}^{n}(1+\theta_i)^2 D_i''} < 0, \tag{7}$$

and so we can establish that if the demand functions are concave, the tariff rate of one importing country is also a *strategic substitute of the monopoly price* for given values of the state and co-state variables.

With $D_i'' \leq 0$, the numerator and the denominator of (7) are negative since by (5) $(p - c + \lambda_M)$ must be positive. Moreover, it is easy to check that (5) is the standard condition that characterizes the monopoly equilibrium:

marginal revenue equal to marginal cost, now including the user cost of the resource. (5) can be written as

$$\frac{\sum_{i=1}^{n} D_i}{\sum_{i=1}^{n}(1+\theta_i)D_i'} + p = c - \lambda_M,$$

and taking common factor p as

$$MR = p\left(\frac{1}{\varepsilon_{Q,p}} + 1\right) = c - \lambda_M = MC,$$

where $\varepsilon_{Q,p}$ is the elasticity of the aggregate demand function.

The open-loop Nash equilibrium is given by the solution to the following system of $2n+3$ equations.

$$p\theta_i = -\lambda_i, \quad i = 1, ..., n, \tag{8}$$

$$\dot{\lambda}_i = r\lambda_i, \ i = 1, .., n, \tag{9}$$

$$0 = \sum_{i=1}^{n} D_i + (p - c + \lambda_M)\sum_{i=1}^{n}(1+\theta_i)D_i', \tag{10}$$

$$\dot{\lambda}_M = r\lambda_M + c'\sum_{i=1}^{n} D_i, \tag{11}$$

$$\dot{x} = \sum_{i=1}^{n} D_i. \tag{12}$$

In order to calculate the steady state we have to take into account that the different countries can have different backstop prices. In this case, what is going to occur is that the countries are going to leave the market sequentially as the monopoly price reaches its backstop price so that the steady state will be defined by $\theta_i^\infty = \lambda_i^\infty = \lambda_M^\infty = 0, \ i = 1, ..., n, \ p^\infty = c(x^\infty)$ and $D_j(p^\infty) = 0$ where j is the country with the highest backstop price. Then it is straightforward that

Proposition 1 *The governments of the importing countries cannot capture the resource rent using a tariff, in other words, the open-loop Nash equilibrium tariff rate with depletion effects is zero.*

This is an immediate consequence of the fact that the tariff, $p\theta_i$, must increase at a constant rate which is *incompatible* with the fact that ultimately the tariff rate must be zero because of the economic exhaustion of the resource. Notice that the price cannot be zero since at the steady state it is equal to the marginal extraction cost. Thus the unique path that can converge to the steady state requires that $\theta_i = \lambda_i = 0$ throughout the exploitation period of the resource and then the system (8)-(9)-(10)-(11)-(12) yields the standard solution for the monopolistic extraction with depletion effects. As long as this argument does not depend on the cost structure, the result will also be valid when there are no depletion effects. It will be valid as well even if the importing countries cooperate imposing the same tariff rate on the resource importations, i.e., even if the importing country governments sign a *tariff agreement* and independently of whether we assume that they are symmetric or not. It is easy to show that the previous result also applies to the case of a *per unit tariff* since all the analysis for the per unit tariff is identical to the one developed in this section simply substituting $p\theta_i$ by the per unit tariff.

2.2 A Tariff Agreement: The Markov-perfect Nash Equilibrium

Next, we want to investigate whether this last result is a consequence of the equilibrium concept used to solve the game. To do so we propose in this section a game for which it is possible to calculate the stationary Markovian (feedback) strategies. Now we assume that the governments of the importing countries sign an agreement to impose the same *per unit tariff* on the resource importations with the aim of maximizing the discounted present value of the sum of the aggregate consumer's welfare.[6] In order to obtain an analytical solution for the game we also assume that the consumer's gross surplus is given by $U_i(q_i) = aq_i - (1/2)q_i^2$ and that the extraction cost is linear, $c(x) = cx$. With these changes we have a differential game between a monopolist and a coalition of the importing country governments that can be written as follows for the monopolist,

$$\max_{\{p\}} \int_0^\infty e^{-rt} \left((p - cx) n(a - p - \theta) \right) dt, \tag{13}$$

and as follows for the coalition of the governments of the importing countries

$$\max_{\{\theta\}} \int_0^\infty e^{-rt} n \left((a-p)(a-p-\theta) - \frac{1}{2}(a-p-\theta)^2 \right) dt, \tag{14}$$

[6] As we have obtained the same qualitative results both for an *ad valorem* tariff and for a per unit tariff, the change in the specification of the tariff does not suppose a strong discontinuity in the analysis developed in this paper. Besides, a per unit tariff allows us to compute the Markov-perfect Nash equilibrium in linear strategies.

the dynamic constraint being

$$s.t.\ \dot{x} = Q = n(a - p - \theta), \quad x(0) = x_0 \geq 0. \tag{15}$$

Markov strategies must satisfy the following system of Hamilton-Jacobi-Bellman equations:

$$\begin{aligned} rW_A &= \max_{\{\theta\}} \left\{ n \left((a-p)(a-p-\theta) - \frac{1}{2}(a-p-\theta)^2 \right) \right. \\ &\quad \left. + W_A' n(a-p-\theta) \right\}, \qquad (16) \\ rW_M &= \max_{\{p\}} \left\{ (p - cx) n(a-p-\theta) + W_M' n(a-p-\theta) \right\}, \qquad (17) \end{aligned}$$

where $W_M(x)$ stands for the optimal current value functions associated with the dynamic optimization problem for the monopoly (13) and $W_A(x)$ for the optimal current value functions associated with the dynamic optimization problem for the agreement (14); i.e., they denote the maxima of the objectives (13) and (14) subject to (15) for the current value of the state variable.

From the first-order conditions for the maximization of the right-hand sides of the HJB equations, we get the instantaneous reaction functions of the governments and the monopolist:

$$\begin{aligned} \theta &= -W_A', \qquad (18) \\ p &= \frac{1}{2}\left(a + cx - W_M' - \theta\right). \qquad (19) \end{aligned}$$

These expressions establish that the optimal tariff is independent of the monopoly price and equal, as before, to the user cost of the resource for the importing countries, and that the price and the tariff are *strategic substitutes* for the monopolist.

By substitution of (18) and (19), we get the solution of the price as a function of the first derivatives of the value functions: $p = \frac{1}{2}\left(a + cx - W_M' + W_A'\right).$ Next, by incorporating the optimal strategies into the HJB Eqs. (16) and (17), we eliminate the maximization and obtain, after some calculations, a pair of nonlinear differential equations:

$$\begin{aligned} rW_A &= \frac{n}{8}(a - cx + W_M' + W_A')^2, \qquad (20) \\ rW_M &= \frac{n}{4}(a - cx + W_M' + W_A')^2. \qquad (21) \end{aligned}$$

In order to derive the solution to this system of differential equations, we guess quadratic representations for the value functions W_A and W_M,

$$W_A(x) = \frac{1}{2}\alpha_A x^2 + \beta_A x + \mu_A,\ W_M(x) = \frac{1}{2}\alpha_M x^2 + \beta_M x + \mu_M, \qquad (22)$$

and we apply the same procedure as the one used by Wirl and Dockner (1995) to calculate the coefficients, see Appendix A. Substituting these coefficients in (18) and (19), we obtain the linear Markov-perfect Nash equilibrium strategies for the tariff and the price:

$$\theta = \frac{an\delta}{4r + 3n\delta} - \frac{n\delta^2}{4r}x, \qquad (23)$$

$$p = \frac{2a(r + n\delta)}{4r + 3n\delta} + \frac{2c + \delta}{6}x, \qquad (24)$$

where

$$\delta = \frac{2}{3n}\left(\left(3cnr + r^2\right)^{0.5} - r\right) > 0.$$

By visual inspection it can be seen that the tariff is inversely related to the accumulated extractions whereas the price increases with the exploitation of the resource. Now using the equilibrium strategies, differential equation (15) can be solved, yielding

$$x = (x_0 - \frac{a}{c})\exp\left\{-\frac{n\delta}{2}t\right\} + \frac{a}{c}, \qquad (25)$$

and by substitution in the equilibrium strategies the tariff and price dynamics are obtained.[7]

$$\theta = \frac{an\delta^2}{4rc}\exp\left\{-\frac{n\delta}{2}t\right\},\ p = a\left(1 - \frac{2c + \delta}{6c}\exp\left\{-\frac{n\delta}{2}t\right\}\right). \qquad (26)$$

Finally, the consumer price can be simply calculated by the addition of the monopoly price and tariff.

$$\pi = \theta + p = a\left(1 - \frac{\delta}{2c}\exp\left\{-\frac{n\delta}{2}t\right\}\right) \qquad (27)$$

so we can summarize these results as

[7] In order to simplify the presentation we assume that $x_0 = 0$. This does not change the sign of the dynamics of these two variables.

Remark 1 *The Markov-perfect Nash equilibrium tariff rate decreases throughout the exploitation period of the resource and converges to zero in the long run. Moreover, the monopoly and consumer prices are increasing and converge to the backstop price.*

Clearly, these results establish that the commitment that the open-loop Nash equilibrium requires for the importing countries, a commitment for the entire exploitation period of the resource, drastically reduces the possibilities of using a tariff to capture part of the monopoly's rents. In other words, the importing country governments have to play feedback strategies, i.e., to define the optimal policy as a function of the accumulated extractions, in order to be able to impose an advantageous tariff for the consumers.

Finally, we obtain the following value functions for the agreement and the monopoly

$$W_A(x) = \frac{n\delta^2}{8r}x^2 - \frac{an\delta}{4r+3n\delta}x + \frac{2a^2nr}{(4r+3n\delta)^2}, \ W_M(x) = 2W_A(x). \tag{28}$$

3 The Case of a Quantity-setting Monopolist

Until now we have assumed that the monopoly chooses the price and the market establishes the resource extraction rate. In this section we analyze the other possibility the monopoly has: to choose the quantity and leave the market to set up the price. In this case by substitution of the inverse demand function in the instantaneous consumer's welfare the following expression is obtained

$$W_i = U_i\left(D_i\left(p(Q,\bar{\theta})\left(1+\theta_i\right)\right)\right) - p(Q,\bar{\theta})D_i\left(p(Q,\bar{\theta})(1+\theta_i)\right), \tag{29}$$

where $\bar{\theta}$ is the vector of the tariff rates.

As the extraction rate is determined by the monopolist, the governments of the importing countries have no influence on the dynamics of the stock. For this reason, in this case the tariff rate is given by the Nash equilibrium of the static game defined by (29). In other words, at each point in time the importing countries choose a tariff rate to maximize the instantaneous flow of the consumer's welfare given the extraction rate and the rival's tariff rates.

The first order conditions yield

$$(U_i' - p)D_i'\left(\frac{\partial p}{\partial \theta_i}(1+\theta_i) + p\right) = \frac{\partial p}{\partial \theta_i}D_i, \ \ i = 1, ..., n,$$

and as $U_i' = p(1+\theta_i)$ the following expression is obtained

$$p\theta_i D_i' \left(\frac{\partial p}{\partial \theta_i}(1+\theta_i) + p \right) = \frac{\partial p}{\partial \theta_i} D_i.$$

By differentiation of the demand function we get

$$\frac{\partial p}{\partial \theta_i} = -\frac{D_i' p}{\sum_{j=1}^{n} D_j'(1+\theta_j)}$$

that by substitution into the above expression yields

$$p\theta_i = -\frac{D_i}{\sum_{j\neq i} D_j'(1+\theta_j)}, \quad i = 1, ..., n. \tag{30}$$

This is the version for an "ad valorem" tariff of the one obtained by Karp and Newbery (1991, p. 288) for a per unit tariff.

Assuming that the system (30) has a unique solution, the dynamic of the extraction rate and hence the dynamics of the tariff rate can be calculated as the solution of a standard optimal control problem.[8] This result shows that the nature of the game changes when the monopoly sets the quantity instead of the price. Now at each moment, given the extraction rate, the importing countries can use the tariffs to reduce the monopoly price and in this way, increase domestic welfare.

3.1 A Tariff Agreement: The Markov-Perfect Stackelberg Equilibrium

In order to complete the analysis of the previous section we look now at the game between the monopolist and the coalition of the importing countries.

When the monopolist chooses the extraction rate, the monopoly price depends on the tariff selected by the countries in the tariff agreement according to the demand inverse function

$$p = a - \theta - (Q/n), \tag{31}$$

so that the instantaneous aggregate welfare of the importing country consumers is written as

$$W_A = n\left(a\frac{Q}{n} - \frac{1}{2}\left(\frac{Q}{n}\right)^2 - \left(a - \theta - \frac{Q}{n}\right)\frac{Q}{n} \right).$$

[8] For a linear demand the existence of a solution could be shown at least for the symmetric case although not the uniqueness. For a per unit tariff both the existence and the uniqueness can be shown.

From this expression it is pretty obvious that the optimal policy is to choose, for a given quantity, a tariff such that the price be zero: $\theta = a - (Q/n)$. However, in this case, the monopoly has no interest in exploiting the resource so that finally in the open-loop Nash equilibrium of the game, the importing countries are not going to obtain any surplus. Given this result and the structure of the game, we think that a Stackelberg equilibrium better represents the relationship between the countries in the tariff agreement and the monopolist. What happens is that the influence of the tariff rate on the monopoly price gives a strategic advantage to the countries in the tariff agreement so that they can behave as a *leader*.[9] Next, we show that the importing countries are interested in establishing a tariff that gives the monopolist the possibility of obtaining a positive price with the aim of obtaining a positive surplus for their consumers. A policy that is clearly superior to the one established above.

Since it is well known that an open-loop Stackelberg equilibrium besides not being subgame perfect (strong time inconsistency) can also be temporally inconsistent (weak time inconsistency),we propose in this section to calculate a Markov-perfect Stackelberg equilibrium which will satisfy the weak time consistency as well. The method of obtaining a Markov-perfect Stackelberg equilibrium we use in this paper was first proposed by Simaan and Cruz (1973). The method is for the leader to treat the follower's HJB equation as a constraint, and to solve his own problem using dynamic programming.

In order to calculate this kind of equilibrium we need the instantaneous reaction function of the follower, i.e., of the monopolist, which is obtained from the following HJB equation where the price is given by (31)

$$rW_M = \max_{\{Q\}} \left\{ (a - \theta - (Q/n) - cx)Q + W_M' Q \right\}.$$

The first-order condition for the maximization of the right-hand side of this equation yields

$$Q = \frac{n}{2} \left(a - \theta - cx + W_M' \right), \tag{32}$$

the monopoly's instantaneous reaction function.

Then the HJB equation for the coalition of the importing country gov-

[9] Lewis, Lindsey and Ware (1986) have analyzed the interaction between a resource monopolist and a coalition of consumers that act collectively to introduce a durable long-lived substitute. They compare the equilibrium predictions of a non-commitment model with two other models where the monopolist and the resource consumer act as time-committed Stackelberg leaders.

ernments can be written as

$$\begin{aligned} rW_A &= \max_{\{\theta\}} \left\{ \frac{n}{8} \left((a - cx + W_M')^2 + 2\,(a - cx + W_M')\,\theta - 3\theta^2 \right) \right. \\ &\quad \left. + W_A' \frac{n}{2} (a - \theta - cx + W_M') \right\}, \qquad (33) \end{aligned}$$

where (31) and (32) have been used to calculate the monopoly price and resource importations $q = Q/n$. The first-order condition for the maximization of the right-hand side of this equation yields the optimal policy or strategy for the tariff which allows us to calculate the optimal policy for the quantity using (32)

$$\theta = \frac{1}{3} (a - cx - 2W_A' + W_M'), \qquad (34)$$

$$Q = \frac{n}{3} (a - cx + W_A' + W_M'). \qquad (35)$$

By substitution of the optimal tariff into the HJB equation of the importing country governments and of the tariff and extraction rates into the HJB equation of the monopoly, we eliminate the maximization and obtain, after some manipulations, the following pair of nonlinear differential equations

$$rW_A = \frac{n}{6} (a - cx + W_A' + W_M')^2, \qquad (36)$$

$$rW_M = \frac{n}{9} (a - cx + W_A' + W_M')^2. \qquad (37)$$

Now, proceeding in the same way as in the previous section, we get the linear Markov-perfect Stackelberg equilibrium strategies for the tariff and extraction rates

$$\theta = \frac{3a(r + n\gamma)}{9r + 5n\gamma} - \frac{9cr + 4n\gamma^2}{27r} x, \qquad (38)$$

$$Q = \frac{3an\gamma}{9r + 5n\gamma} - \frac{n\gamma}{3} x, \qquad (39)$$

where

$$\gamma = \frac{3}{10n} \left((20cnr + 9r^2)^{0.5} - 3r \right) > 0. \qquad (40)$$

By visual inspection it can be seen that the tariff and extraction rates are inversely related to the accumulated extractions.

Since $\dot{x} = Q$, we can use (39) to calculate the dynamics of the accumulated extractions for $x_0 = 0$

$$x = \frac{a}{c} \left(1 - \exp\left\{ -\frac{n\gamma}{3} t \right\} \right), \qquad (41)$$

and by substitution in the equilibrium strategies the dynamics of the tariff and extraction rates

$$\theta = \frac{a(9cr + 4n\gamma^2)}{27cr} \exp\left\{-\frac{n\gamma}{3}t\right\}, \quad Q = \frac{an\gamma}{3c} \exp\left\{-\frac{n\gamma}{3}t\right\}. \tag{42}$$

Now by substitution in the demand inverse function the monopoly price can be calculated,

$$p = a\left(1 - \frac{9cr + 9r\gamma + 4n\gamma^2}{27cr} \exp\left\{-\frac{n\gamma}{3}t\right\}\right), \tag{43}$$

and adding this price to the tariff rate, the consumer price is

$$\pi = \theta + p = a\left(1 - \frac{\gamma}{3c} \exp\left\{-\frac{n\gamma}{3}t\right\}\right), \tag{44}$$

so we can summarize these results as

Remark 2 *The Markov-perfect Stackelberg equilibrium tariff and extraction rates decrease throughout the exploitation period of the resource and converge to zero in the long run. Moreover, the monopoly and consumer prices are increasing and converge to the backstop price.*

This result along with the previous one establish that it is advantageous for the importing countries to coordinate and impose a common tariff on the resource importations, both if the monopoly chooses the price and if the monopoly chooses the extraction rate. However, in this second case, the importing countries enjoy a strategic advantage and can impose a higher tariff rate as we show in the next section.

Finally, we obtain the following value functions for the agreement and the monopoly

$$\begin{aligned} W_A(x) &= \frac{n\gamma^2}{6r}x^2 - \frac{3an\gamma}{9r + 5n\gamma}x + \frac{27a^2nr}{2(9r + 5n\gamma)^2}, && (45) \\ W_M(x) &= \frac{n\gamma^2}{9r}x^2 - \frac{2an\gamma}{9r + 5n\gamma}x + \frac{9a^2nr}{(9r + 5n\gamma)^2}. && (46) \end{aligned}$$

4 Comparing the Two Equilibria

This section compares the Nash equilibrium (MPNE) and the Stackelberg equilibrium in which importing countries act as a leader (MPSE). First, we compare the *initial* values of the optimal strategies.

Lemma 1 *The initial consumer price and tariff rate are lower and the initial monopoly price is higher in the MPNE than in the MPSE.*

Proof. See Appendix B. ■

This result establishes that the strategic advantage of the importing country governments translates into a higher initial value for the tariff, which reduces the demand for the resource. The reduction in initial demand explains why the initial monopoly price is lower in the MPSE. Thus, a higher tariff has two effects on the consumer price: one direct and positive and another indirect and negative through the monopoly price. The net effect is positive because the reduction in demand does not completely translate into the monopoly price, given that the demand function is linear and the marginal extraction cost is constant. Hence, the initial consumer price is lower in the MPNE although the monopoly price is higher.

We now turn to the transitional dynamics.

Proposition 2 *The tariff in the MPSE is above the MPNE tariff. Contrarily, the monopoly price in the MPSE is below the MPNE monopoly price. However, the consumer price in the MPSE is first above, but later below, the MPNE consumer price.*

Proof. See Appendix C. ■

This result is a logical consequence of the fact that both equilibria converge to the same long run equilibrium characterized by the economic exhaustion of the resource. Accordingly, the total amount mined is the same – irrespective of the equilibrium concept used to solve the game - and the area under the temporal path of the extraction rate must therefore be the same as well. The temporal paths must thus intersect. The monotonic behavior of the variables explains why the paths intersect only once. The intersection of the temporal paths of the extraction rate occurs along with the intersection of the temporal paths of the consumer price. However, for the tariff and monopoly price there are no intersection points. This is possible because of the inverse relationship between the tariff and monopoly price for both equilibria. In the MPSE the tariff is higher than the tariff in the MPNE whereas the monopoly price is lower. Then as the consumer price is given by the tariff plus the monopoly price, the consumer price can be first higher, and later lower, in the MPSE than in the MPNE.

Moreover, it is easy to show that although the leadership position is advantageous for the importing countries, the efficiency of the market decreases.

Proposition 3 *When the importing country governments have a strategic advantage, the aggregate consumer's welfare increases while the monopoly's rent and aggregate welfare (measured as the aggregate consumer's welfare plus monopoly's rent) decrease, compared with the MPNE.*

Proof. See Appendix D. ■

We show that this result holds for any value of the accumulated extractions less than its steady state level. This is a standard result that can be found in the comparison between the Nash and Stackelberg equilibria in different models.

5 Conclusions

In this paper we have revisited the issue, first tackled by Bergstrom (1982), of using a tariff on a non-renewable resource importations in order to appropriate part of the monopoly's rents. We extend the analysis taking into account that the exploitation of non-renewable resources is characterized by the presence of depletion effects, i.e., the marginal extraction cost increases for the same extraction rate as the accumulated extractions increase.

For the case of a monopolistic market the results depend crucially on the kind of strategies the importing country governments play and on whether the monopolist chooses the price or the extraction rate. For a price-setting monopolist we show that the importing countries cannot use a tariff to capture the monopoly rents if they are constrained to use open-loop strategies, i.e., if they commit to a temporal path for the tariff, even if the governments sign a tariff agreement to impose the same tariff. This result drastically changes if the importing countries co-operate through a tariff agreement and use Markov (feedback) strategies, i.e., if they commit to a rule that fixes the tariff as a function of the accumulated extractions (the state variable of the game). In this case a tariff is clearly advantageous for the consumers of the importing countries. For a quantity-setting monopolist the nature of the game changes, in fact, for the importing country governments the game becomes a static game and now the importing countries find it advantageous to set a tariff on resource importations. Finally, we show that when the governments of the importing countries sign a tariff agreement they enjoy a strategic advantage which allows them to act as the leader of the game.

Although we think that this paper clarifies and extends the analysis of the possibilities of using a tariff to capture non-renewable resource rents it would be of interest to address this issue when there is no cooperation among the importing country governments for the case of a price-setting monopolist.

In particular, we have calculated the Markov-perfect Nash equilibrium for a differential game between a coalition of importing country governments and a monopolist that sets the price but, although we guess that the qualitative result is not going to change, it would be useful to know whether the importing countries can gain by imposing a feedback tariff without coordination, i.e., without signing a tariff agreement. Another issue that we would like to address in the future is the one relating to the stability of the agreements.

A Derivation of the Stationary Linear Markov Strategies

Substituting W_A, W'_A, W_M and W'_M into (20) and (21), collecting terms with equal powers of x, and equating the coefficients of these terms to zero, one obtains the following system of coupled Riccati equations

$$
\begin{aligned}
4r\alpha_A &= n(c-\alpha_A-\alpha_M)^2, & (47)\\
2r\alpha_M &= n(c-\alpha_A-\alpha_M)^2, & (48)\\
4r\beta_A &= -n(a+\beta_A+\beta_M)(c-\alpha_A-\alpha_M), & (49)\\
2r\beta_M &= -n(a+\beta_A+\beta_M)(c-\alpha_A-\alpha_M), & (50)\\
8r\mu_A &= n(a+\beta_A+\beta_M)^2, & (51)\\
4r\mu_M &= n(a+\beta_A+\beta_M)^2. & (52)
\end{aligned}
$$

These equations can be explicitly solved by the following change in the variables: $y=\alpha_A+\alpha_M$ and $z=\beta_A+\beta_M$. Adding the first two equations and the last two equations yields a system in the new variables

$$
\begin{aligned}
4ry &= 3n(c-y)^2, & (53)\\
4rz &= -3n(a+z)(c-y). & (54)
\end{aligned}
$$

The solution for the first equation is

$$y = c + \frac{2r}{3n} \pm \frac{2}{3n}\left(3cnr + r^2\right)^{0.5}. \qquad (55)$$

In order to choose between the two roots a stability condition is used. Next, we develop this stability condition. Using the proposed value functions the linear Markov strategies can be written as

$$\theta = -\alpha_A x - \beta_A, \;\; p = \frac{1}{2}\left(a+\beta_A-\beta_M+(c+\alpha_A-\alpha_M)x\right), \qquad (56)$$

so that the dynamics of the accumulated extractions is given by

$$\dot{x} = \frac{n}{2}\left(a + \beta_A + \beta_M - (c - \alpha_A - \alpha_M)x\right).$$

Then, we have that

$$\frac{d\dot{x}}{dx} < 0 \ \rightarrow \ \frac{d\dot{x}}{dx} = -\frac{n}{2}(c - \alpha_A - \alpha_M) = -\frac{n}{2}(c - y) < 0,$$

so that the stability condition requires that $c - y > 0$. This condition is satisfied by the lowest root of (55) yielding

$$\delta = c - y = \frac{2}{3n}\left(\left(3cnr + r^2\right)^{0.5} - r\right) > 0. \tag{57}$$

With this result α_A and α_M can be obtained directly from (47) and (48)

$$\alpha_A = \frac{n\delta^2}{4r}, \ \ \alpha_M = \frac{n\delta^2}{2r}. \tag{58}$$

Next, we calculate z using (54)

$$z = -\frac{3an\delta}{4r + 3n\delta} < 0,$$

and then β_A and β_M from (49) and (50)

$$\beta_A = -\frac{an\delta}{4r + 3n\delta} < 0, \ \ \beta_M = -\frac{2an\delta}{4r + 3n\delta} < 0. \tag{59}$$

By substitution in (56) we obtain the linear Makov-perfect Nash equilibrium strategies for the tariff and the price (23) and (24). Finally, using (59) in (51) and (52) we obtain

$$\mu_A = \frac{2a^2nr}{(4r + 3n\delta)^2}, \ \ \mu_M = \frac{4a^2nr}{(4r + 3n\delta)^2},$$

and by substitution the value functions (28).

B Proof of Lemma 1

Let us suppose that $\theta^S(0) \leq \theta^N(0)$.[10] Then using (26) and (42) for $t = 0$ we obtain after obvious simplifications that $36cr + 16n\gamma^2 \leq 27n\delta^2$. In Appendix

[10]Superscript N stands for the MPNE and S for the MPSE.

A we have establish that $4ry^N = 3n(c-y^N)^2 = 3n\delta^2$, see (53). On the other hand, the Riccati equations for the MPSE yield $9ry^S = 5n(c-y^S)^2 = 5n\gamma^2$ where $\gamma = c - y^S$ by definition. Then by substitution of $n\gamma^2$ and $n\delta^2$ in the above inequality we obtain that $5\delta + y^S \leq 0$. Developing $9ry^S = 5n(c-y^S)^2$ we obtain the following quadratic equation $(y^S)^2 - (2c + (9r/5n))y^S + c^2 = 0$ which has two positive roots, the lowest root being the one that satisfies the stability condition so that $\gamma = c - y^S > 0$. Then as δ is positive, see (57) and y^S as well, we have gotten a contradiction $5\delta + y^S \leq 0$, and $\theta^S(0) > \theta^N(0)$ is established.

Next, we compare the initial monopoly prices. Let us suppose that $p^S(0) \geq p^N(0)$. Then using (26) and (43) for $t = 0$ we obtain after obvious simplifications that $54r\gamma + 24n\gamma^2 \leq 27r\delta$. Using again that $9ry^S = 5n\gamma^2$ we obtain after substituting $n\gamma^2$ in the previous inequality and rearranging terms that $81c + 54\gamma + 135y^N \leq 0$. Where y^N is the lowest root of (55). It is very easy to show that this root is positive so that a contradiction is established since γ is also positive. Then we have that $p^S(0) < p^N(0)$. Finally, we compare the initial consumer prices. Using (27) and (44) we have that

$$\pi^S(0) - \pi^N(0) = \frac{a}{c}\left(\frac{\delta}{2} - \frac{\gamma}{3}\right),$$

which yields by substitution of δ and γ

$$\pi^S(0) - \pi^N(0) = \frac{a}{c}\left(\frac{c}{6} + \frac{y^S}{3} - \frac{y^N}{2}\right),$$

and now by substitution of y^S and y^N

$$\pi^S(0) - \pi^N(0) = \frac{a}{c}\left(\frac{1}{3n}(3cnr + r^2)^{0.5} - \frac{r}{30n} - \frac{1}{10n}(20cnr + 9r^2)^{0.5}\right).$$

Let us suppose that this difference is negative or zero. Then we can write reordering terms and simplifying

$$10(3cnr + r^2)^{0.5} \leq r + 3(20cnr + 9r^2)^{0.5}.$$

Squaring, reordering terms and squaring again we have the contradiction: $4cn + r \leq 0$. Thus, we obtain that $\pi^S(0) > \pi^N(0)$, which also implies that $\delta/2 - \gamma/3$ is positive.

C Proof of Proposition 2

For the comparison of the tariff temporal paths, we use (26) and (42). The difference between the two temporal paths is given by

$$\theta^S - \theta^N = \theta^S(0)\exp\left\{-\frac{n\gamma}{3}t\right\} - \theta^N(0)\exp\left\{-\frac{n\delta}{2}t\right\}.$$

For $t = 0$ we know from Lemma 1 that the difference $\theta^S(0) - \theta^N(0)$ is positive. For $t \neq 0$ we can find the number of intersection points from the equation $\theta^S - \theta^N = 0$, which can be written as

$$\frac{\theta^N(0)}{\theta^S(0)} = \exp\left\{n\left(\frac{\delta}{2} - \frac{\gamma}{3}\right)t\right\}.$$

However, this equation has no solution for $t \geq 0$ since the l.h.s. is a positive constant less than one and the r.h.s. is an increasing and convex function which takes the unit value for $t = 0$, and tends to infinity when t tends to infinity since as it has been shown in Lemma 1 $\delta/2 - \gamma/3$ is positive. Hence, the temporal path of the MPSE tariff is above the temporal path of the MPNE in the interval $[0, \infty)$. The same procedure can be used to show that the temporal path of the MPSE monopoly price is below the temporal path of the MPNE in the interval $[0, \infty)$. For comparing the temporal paths of the consumer price we calculate the difference between the two temporal paths using (27) and (44)

$$\pi^S - \pi^N = \frac{a\delta}{2c}\exp\left\{-\frac{n\delta}{2}t\right\} - \frac{a\gamma}{3c}\exp\left\{-\frac{n\gamma}{3}t\right\},$$

and we can find the number of intersection points from the equation $\theta^S - \theta^N = 0$ given by

$$\frac{\delta/2}{\gamma/3} = \exp\left\{n\left(\frac{\delta}{2} - \frac{\gamma}{3}\right)t\right\}, \tag{60}$$

where the l.h.s. is a positive constant higher than one and the r.h.s. is an increasing and convex function which takes the unit value for $t = 0$, and tends to infinity when t tends to infinity as we have just seen. Hence, the temporal paths cut each other once in the interval $[0, \infty)$, and consequently, for $0 \leq t < t'$, where t' is the solution to Eq. (60), the MPSE consumer price is above the MPNE consumer price, whereas for $t' < t$ the relationship is the contrary.

D Proof of Proposition 3

We begin this proof comparing the aggregate consumer's welfare. First, we show that the value functions reach an absolute minimum for $x = a/c$ for which we have that $W_A^S(a/c) = W_A^N(a/c) = 0$. This implies that the value functions (28) and (45) are positive and decreasing in the interval $[0, a/c)$.The first order condition $dW_A^N(x)/dx = 0$ yields

$$\frac{n\delta^2}{4r}x^{N^*} - \frac{an\delta}{4r + 3n\delta} = 0$$

whose solution is

$$x^{N^*} = \frac{4ar}{4r\delta + 3n\delta^2}.$$

According to (57) the denominator can be written as $4rc - 4ry^N + 3n(c - y^N)^2$ where $-4ry^N + 3n(c - y^N)^2 = 0$ according to (53) which yields $x^{N^*} = 4ar/4rc = a/c$. Now, by substitution we obtain that

$$W_A^N(a/c) = \frac{a^2n((4r\delta + 3n\delta^2)^2 - 16r^2c^2)}{8rc^2(4r + 3n\delta)^2} = 0.$$

This expression is zero since, as we have just shown, $4r\delta + 3n\delta^2 = 4rc$. This establishes that the value function is positive and decreasing in the interval $[0, a/c)$. The same kind of arguments apply for $W_A^S(x)$.The first order condition $dW_A^S(x)/dx = 0$ yields

$$\frac{n\gamma^2}{3r}x^{S^*} - \frac{3an\gamma}{9r + 5n\gamma} = 0$$

whose solution is

$$x^{S^*} = \frac{9ar}{9r\gamma + 5n\gamma^2}.$$

According to what we have written in Appendix B the denominator can be written as $9rc - 9ry^S + 5n(c - y^S)^2$ where $-9ry^S + 5n(c - y^S)^2 = 0$, see also Appendix B, which yields $x^{S^*} = 9ar/9rc = a/c$. Now, by substitution we obtain that

$$W_A^S(a/c) = \frac{a^2n((9r\gamma + 5n\gamma^2)^2 - 81r^2c^2)}{6rc^2(9r + 5n\gamma)^2} = 0.$$

This expression is zero since $9r\gamma + 5n\gamma^2 = 9rc$. This establishes that the value function for the agreement in the MPSE is positive and decreasing in the interval $[0, a/c)$.

Next, we calculate the difference

$$W_A^S(x) - W_A^N(x) = \left(\frac{n\gamma^2}{6r} - \frac{n\delta^2}{8r}\right)x^2 - \left(\frac{3an\gamma}{9r+5n\gamma} - \frac{an\delta}{4r+3n\delta}\right)x + \frac{27a^2nr}{2(9r+5n\gamma)^2} - \frac{2a^2nr}{(4r+3n\delta)^2}. \quad (61)$$

The first coefficient of this linear-quadratic function is positive if $4\gamma^2 - 3\delta^2$ is positive. By substitution of γ and δ, see (40) and (57), we obtain

$$\frac{1}{n^2}\left[\frac{9}{25}((20cnr + 9r^2)^{0.5} - 3r)^2 - \frac{4}{3}((3cnr + r^2)^{0.5} - r)^2\right].$$

Let us suppose that $4\gamma^2 - 3\delta^2 \leq 0$.Then it must be satisfied that

$$\frac{9}{25}((20cnr + 9r^2)^{0.5} - 3r)^2 \leq \frac{4}{3}((3cnr + r^2)^{0.5} - r)^2.$$

Taking the positive square root and reordering we obtain

$$3\sqrt{3}(20cnr + 9r^2)^{0.5} \leq 10(3cnr + r^2)^{0.5} + 5.59r.$$

Squaring, reordering terms and squaring again we get the contradiction: $57600c^2n^2r^2 + 16172cnr^3 \leq 0$ which establishes that $4\gamma^2 - 3\delta^2$ is positive. Now, we look at the second coefficient of the difference $W_A^S(x) - W_A^N(x)$. This second coefficient is positive if $4\gamma - 3\delta$ is positive. By substitution we get now

$$\frac{2}{n}\left[\frac{6}{10}((20cnr + 9r^2)^{0.5} - 3r) - (3cnr + r^2)^{0.5} + r\right].$$

Let us suppose that $4\gamma - 3\delta \leq 0$. Then it must be satisfied that

$$3(20cnr + 9r^2)^{0.5} \leq 5(3cnr + r^2)^{0.5} + 4r.$$

Squaring, reordering terms and squaring again we get the contradiction: $11025c^2n^2 + 3600cnr \leq 0$ which establishes that $4\gamma - 3\delta$ is positive. Finally, we compare the independent coefficients. Let us suppose that

$$\frac{27a^2nr}{2(9r+5n\gamma)^2} - \frac{2a^2nr}{(4r+3n\delta)^2} \leq 0.$$

Developing this inequality we obtain

$$432r^2 + 648rn\delta + 243n^2\delta^2 \leq 324r^2 + 360rn\gamma + 100n^2\gamma^2,$$

which is a contradiction if $\delta > \gamma$. Let us suppose that $\delta \leq \gamma$. Then it must be satisfied that

$$20(3cnr + r^2)^{0.5} + 7r \leq 9(20cnr + 9r^2)^{0.5}.$$

Squaring and reordering terms we get the following contradiction:

$$368r^2 + 1020cnr + 280r(3cnr + r^2)^{0.5} \leq 0.$$

Thus, we can conclude that $\delta > \gamma$ which establishes that

$$\frac{27a^2nr}{2(9r + 5n\gamma)^2} > \frac{2a^2nr}{(4r + 3n\delta)^2}$$

in (61). This means that all the coefficients in (61) are positive and that this difference presents an absolute minimum given by the first order condition

$$\left(\frac{n\gamma^2}{3r} - \frac{n\delta^2}{4r}\right)x^* - \left(\frac{3an\gamma}{9r + 5n\gamma} - \frac{an\delta}{4r + 3n\delta}\right) = 0, \tag{62}$$

but we know that

$$\begin{aligned}
\frac{dW_A^S(x)}{dx} &= 0 \to \frac{n\gamma^2}{3r}x^{S*} - \frac{3an\gamma}{9r + 5n\gamma} = 0 \to x^{S*} = \frac{a}{c}, \\
\frac{dW_A^N(x)}{dx} &= 0 \to \frac{n\delta^2}{4r}x^{N*} - \frac{an\delta}{4r + 3n\delta} = 0 \to x^{N*} = \frac{a}{c},
\end{aligned}$$

so that the unique solution for (62) is a/c. This allows us to conclude that the difference $W_A^S(x) - W_A^N(x)$ is positive and decreasing for x in the interval $[0, a/c)$. In other words, when the importing country governments have a strategic advantage, the aggregate consumer's welfare in the MPSE is greater the aggregate consumer's welfare in the MPNE.

The comparison between $W_M^S(x)$ and $W_M^N(x)$ and also the comparison between aggregate welfare $W_A^S(x) + W_M^S(x)$ and $W_A^S(x) + W_M^S(x)$ follow step by step the comparison we have just finished to present for this reason we omit them. We only mention that the value functions for the monopoly in both equilibria have the same properties that the value functions for the agreement. They are positive and decreasing in the interval $[0, a/c)$.

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